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From Piston Theory to a Unified Hypersonic–Supersonic Lifting Surface Method

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A unified hypersonic–supersonic lifting surface method has been developed, where the concept of piston theory is generalized and suitably integrated with the aerodynamic influence coefficient (AIC) matrix due to linear theory. Thus, this unified method can account for the effects of wing thickness and/or flow incidence, upstream influence, and three dimensionality for an arbitrary lifting surface system in an unsteady flow, whereas piston theory fails to account for the latter effects. In particular, the present composite series renders the AIC matrix uniformly valid for all supersonic–hypersonic Mach numbers, thus extending the method applicability to cover both the Ackeret limit at the low supersonic end and the Newtonian limit at the hypersonic end. From various cases studied it is concluded that the present method makes a substantial improvement over the linear lifting surface theory and piston theory in terms of unsteady pressures, stability derivatives, and flutter speeds. Among other theories it also predicts the most conservative flutter boundary and it confirms that the supersonic thickness effect is to reduce the flutter speed.

Introduction

HAYES and Lighthill's piston theory[1,2] has been one of the most commonly practiced methods in supersonic aeroelastic applications. Because of its simplicity and inclusion of nonlinear thickness/incidence effect, nearly all supersonic panel flutter analyses in recent years have adopted the piston theory as their aerodynamic model. Its ease of application and its acceptable accuracy render the theory an effective tool for many aeroelastic problems.[3]

However, two inherent undesirable features of Lighthill's piston theory[2] (hereafter called piston theory) invalidate its capability in general aeroelastic applications. First, the theory is a strictly one-dimensional, quasi-steady theory, whereby no upstream influence nor flow history could be accounted for. Second, its domain of application usually covers a restrictive range of Mach numbers, depending on the thickness and frequency parameters given.

Within the last decade, exact three-dimensional linear theory has been sufficiently developed for treatments of lifting surfaces in unsteady supersonic flow.[4] Nevertheless, supersonic lifting surface methods, such as the harmonic gradient method (HGM, or known as the ZONA51 code),[4] are confined to planforms of very thin sections, whereby no thickness effect can be accounted for. On the other hand, it has been known for some time that the supersonic thickness effect could render a forward shift in the aerodynamic center, thereby reducing the flutter speed.

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In view of the recent development of the National Aerospace Plane (NASP) and High Speed Civil Transport (HSCT), a general supersonic flutter method that could account for the thickness effect of wing sections would be very desirable. The objective of this paper is to present our recent development of such a lifting surface method that generalizes linear theory to include the effects of nonlinear thickness and upstream influence in a unified supersonic–hypersonic flow regime.

Piston Theory

Subsequent to the original publication of Lighthill,[2] Ashley and Zartarian[3] first proposed the application of the piston theory for flutter analysis and other aeroelastic applications. They found that the nonlinear thickness effect provided by the theory indeed results in a more conservative flutter boundary, which was validated by measured data. Based on a criterion that if any one of the conditions holds, namely

$$M^2 >> 1, \qquad kM^2 >> 1 \quad \text{or} \quad k^2M^2 >> 1 \tag{1}$$

Landahl et al.[5] further established a consistent linearized piston theory, where M is the freestream Mach number, k is the reduced frequency defined as $k = \omega c/U_\infty$, ω is the circular frequency, c is the chord length, and U_∞ is the freestream velocity. With this theory, they obtained an explicit flutter solution for a typical two-dimensional wing section. The flutter speed according to their theory approaches those predicted by the exact linear theory[6] as the Mach number increases, whereas it tends to depart from the latter as the Mach number decreases toward unity.

Originally, Lighthill's piston theory[2] accounts for the effect of nonlinear thickness in the high Mach number range such that $M^2 >> 1$. It imposes the condition that the magnitude of the piston velocity never exceeds the speed of sound in the undisturbed fluid. The aerodynamics of this analogy is to state that

$$M\tau < 1 \quad \text{and} \quad kM\tau < 1 \tag{2}$$

where τ is the thickness or the flow incidence of the airfoil, whichever is larger. In terms of Tsien's hypersonic similarity parameter K (Ref. 7), where $K = M\tau$, inequality (2) is to say that both K and kK should be bound by unity. While the condition $K < 1$ for piston theory may be somewhat relaxed to include regions near $K \approx 1.0$, the condition $kK < 1$ of the inequality (2) is a rather stringent one. Thus, in an unsteady hypersonic flow, if $K \approx \mathcal{O}(1)$, then k must be kept very small. The failure of piston theory in the moderate to high range of k is evidenced by the panel flutter results presented in the work of Chavez and Liu.[8]

Third-Order Theories

Following the suggestion of Morgen et al.,[9] Rodden et al.[10] arrived at a generalized expression for the pressure coefficients, i.e.,

$$Cp = \frac{2}{M^2}\left[c_1\left(\frac{w}{a_0}\right) + c_2\left(\frac{w}{a_0}\right)^2 + c_3\left(\frac{w}{a_0}\right)^3\right] \tag{3}$$

where w represents the piston upwash.

For piston theory[2]

$$c_1 = 1, \qquad c_2 = (\gamma + 1)/4 \quad \text{and} \quad c_3 = (\gamma + 1)/12 \tag{4}$$

For Van Dyke's second-order theory[11]

$$c_1 = \frac{M}{m} \quad \text{and} \quad c_2 = \frac{M^4(\gamma + 1) - 4m^2}{4m^4} \tag{5}$$

where $m^2 = M^2 - 1$, and γ is the ratio of specific heats.

A modified piston theory is recommended[9] to replace c_1 and c_2 of Eq. (4) by that of Eq. (5), rendering an extension to the lower Mach number region (referred to hereinafter as Van Dyke's theory[11]).

The c_1 and c_2 of Van Dyke[11] were first obtained by Busemann,[12] in which he also included a third-order term based on a consistent expansion of the simple wave theory, i.e.,

$$c_3 = (1/6m^7)(a_0M^8 + b_0M^6 + c_0M^4 + d_0M^2 + e_0) \tag{6}$$

where

$$a_0 = \gamma + 1, \qquad b_0 = 2\gamma^2 - 7\gamma - 5$$

$$c_0 = 10(\gamma + 1), \qquad d_0 = -12 \quad \text{and} \quad e_0 = 8$$

Following Busemann,[12] Donov[13] further developed a comprehensive theory in which he obtains series expansion solution up to the fourth-order term, accounting separately for the isentropic part and the rotational part due to simple wave and shock wave, respectively. Also derived independently by Carafoli,[14] Donov's third-order term[13] including the effect of shock wave reads

$$c_3 = (1/6Mm^7)(aM^8 + bM^6 + cM^4 + dM^2 + e) \tag{7}$$

where

$$a = 3\left(\frac{\gamma + 1}{4}\right)^2, \qquad b = \frac{3\gamma^2 - 12\gamma - 7}{4}$$

$$c = \frac{9(\gamma + 1)}{2}, \qquad d = -6 \quad \text{and} \quad e = 4$$

In passing, it is noted that through a different approach, Kahane and Lees[15] have obtained a correction term to c_3 of Eq. (6), resulting in essentially the same c_3 as that of Eq. (7).

From the previous analysis, therefore, a consistent choice of Cp would be to adopt Donov's series[13] and Busemann's series[12] for flow compression and expansion, respectively.

For unsteady flow applications, Eq. (3) is recast into the form of pressure differential of the upper and the lower wing surfaces, i.e., $\Delta Cp = Cp_{\text{lower}} - Cp_{\text{upper}}$, and the piston velocity w/U_∞ is represented by two terms, i.e., $w/U_\infty = w_0 + w_1$, where w_0 denotes the slope of the thickness distribution of the wing section and w_1 denotes the downwash. Thus, the total pressure differential $\Delta\bar{C}p$ can be expressed as

$$\Delta\bar{C}p = \Delta Cp_0 + \Delta Cp \tag{8}$$

and up to the third-order term

$$\Delta Cp_0 \cong \frac{2}{M^2}\sum_{n=1}^{3} c_nM^n(\Delta w_0)^{(n)} \tag{8a}$$

$$\Delta Cp \cong \frac{4w_1}{M^2}\sum_{n=1}^{3} nc_nM^n(\Delta\bar{w}_0)^{(n-1)} + [6w_1^2\Delta w_0^{(1)} + 4w_1^3]c_3M \tag{8b}$$

where

$$(\Delta w_0)^{(n)} \equiv w^n_{0_{\text{lower}}} - w^n_{0_{\text{upper}}} \quad (\Delta\bar{w}_0)^{(n)} \equiv \frac{w^n_{0_{\text{lower}}} + w^n_{0_{\text{upper}}}}{2} \tag{8c}$$

For nonlifting airfoil sections, where $(w_0)_l = (w_0)_u$, Eq. (8b) reduces to the expression

$$\Delta Cp = (4/M)[(c_1 + 2c_2Mw_0 + 3c_3M^2w_0^2)w_1 + (c_3M^2)w_1^3] \tag{9}$$

which includes the third-order piston theory as a special case.

Hypersonic Similitude

A classical hypersonic similarity[16] can be expressed as

$$C_p = (2/M^2)fn(K, \gamma) \tag{10}$$

where

$$fn = \frac{1}{\gamma}\left[\left(1 + \frac{\gamma - 1}{2}K\right)^{2\gamma/(\gamma-1)} - 1\right] \tag{10a}$$

is the universal function due to the Prandtl–Meyer expansion, and

$$fn = K^2\left\{\left[\left(\frac{\gamma + 1}{4}\right)^2 + \frac{1}{K^2}\right]^{1/2} + \frac{\gamma + 1}{4}\right\} \tag{10b}$$

is the universal function due to oblique shock waves, where $K = M\tau$.

Clearly, Eq. (10a) is the basis of Lighthill's piston theory[2] and, hence, of Eq. (3). Equation (10b) was established by Tsien[7] and Linnell.[17] When Eq. (10b) is expanded up to the third-order term, the coefficient c_3 corresponding to Eq. (4) reads

$$c_3 = (\gamma + 1)^2/32 \tag{11}$$

This is to say that the departure between Eqs. (10a) and (10b) starts from the third-order term and the difference of which amounts to $\Delta c_3 = (3\gamma^2 - 2\gamma - 5)/96 < 0$, representing the difference in rotationality due to shock wave.

It is desirable to extend the previous third-order theories into the hypersonic flow regime where $K \geq \mathcal{O}(1)$. Close examination of these theories reveals that the Cp's of these third-order theories diverge drastically as K increases toward the Newtonian limit (see Fig. 1). While Van Dyke's second-order theory[11] results in a Cp that is one-half the value of Newtonian pressure, the Cp of linear theory vanishes at the Newtonian limit. It is clear that piston theory does not yield the correct

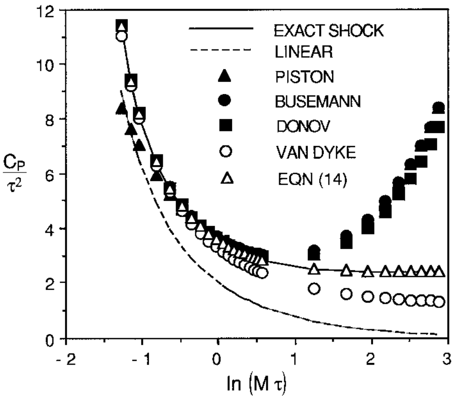

Fig. 1 Surface pressure of a wedge according to various supersonic/hypersonic models: τ = tan(10 deg), γ = 1.4.

limit in the low supersonic end, nor does it approach the Newtonian limit in the hypersonic end. Figure 1 shows that piston theory has a limited valid range of $\ell n\ K$ between about -1 to at the most 0.5 ($0.368 < K < 1.05$), for a wedge with a semiangle of 10 deg.

Clearly, the status of the previous third-order theories warrants further establishment of one that is uniformly valid and covers both the supersonic and hypersonic limits. In the present development we have established such a uniformly valid solution by means of a strained parameter technique[18] in the unified supersonic–hypersonic domain. The resulting Cp's are two composite functions, one for the compression waves and the other for the expansion waves, which can be generally recast into a pseudosimilar form as

$$Cp/\tau^2 = fc(K, \gamma;\ c_1, c_2, c_3, fn) \tag{12}$$

The coefficients c_1, c_2, and c_3 are suitably chosen from the appropriate third-order theories, and the function fn is referred to those of Eqs. (10a) and (10b). Equation (12) will be discussed further in the next section.

Unified Lifting Surface Method

Recall the classical lifting surface formulation[19] in which the supersonic integral equation results at the wing surface:

$$\left(\frac{\partial}{\partial x} + ik\right) h = \int\int_A \Delta Cp(\xi, \eta)\cdot K(x - \xi, y - \eta, 0)\ d\xi\ d\eta \tag{13}$$

$$K = -\frac{1}{2\pi}\left\{\frac{\partial^2}{\partial z^2}\int\int_{R=0}^{x-\xi}\frac{\cos(kMR/m^2)}{R}\exp(-ik\lambda/m^2)\ d\lambda\right\}_{z=0}$$
$$\times \exp[-ik(x-\xi)] \qquad R = \sqrt{\lambda^2 - m^2[(y-\eta)^2 + z^2]} \tag{13a}$$

where $h = h(x, y)$ is the given mode shape.

Equation (13) is recast into a matrix equation where the downwash matrix $[D]$ due to the kernel integral K is related to the downwash function w_1 as

$$[D]\{\Delta Cp\} = \{w_1\} \tag{13b}$$

where $w_1 = h_x + ikh$, and $\Delta Cp = \Delta Cp(x, y;\ k, \tau = 0)$.

The thickness correction of the previous linearized ΔCp solution is proceeded as follows. Applying amplitude perturbation upon the exact tangency condition and further ignoring the thickness–modal interaction on the downwash terms therein, there is obtained an approximated unified downwash matrix equation

$$([D] + \mu\cdot[E])\{\Delta Cp\} \approx \{w_1\} \tag{14}$$

where the matrix $[E]$ is nonlinear in that it contains nonlinear terms in thickness τ or in flow incidence α_0. Clearly, the unsteady upstream influence along with the flow three dimensionality are provided by matrix $[D]$; and the quasi-steady upstream influence due to thickness is by matrix $[E]$, if it were derived from the third-order theories [i.e., Eq. (9) or Eq. (12)]. Thus, the resulting pressure reads $\Delta Cp = \Delta Cp(x, y;\ k, \tau, \tau^2)$.

There are a number of alternative approaches that can provide the $[E]$ matrix. For example, a stripwise solution can be provided by the perturbed Euler characteristic (PEC) method.[8] In passing, we note that the formulation of the PEC method is a generalization of Hui's exact theory,[20] in that the low-frequency assumption is removed, where the unsteady and thickness upstream influences are fully included. Thus, in some subsequent cases, the PEC solutions are used as the benchmarks for result verification. Nonetheless, employing the PEC method to construct the $[E]$ matrix could be rather involved. The simplest and most expedient approach, by comparison, is to adopt the previous third-order theories. In so doing, it is required that $[E]$ be a diagonal matrix whose inverse $[E]^{-1}$ contains elements that are related to two nonlinear functions in w_0, i.e.,

$$E_{ii}^{-1} = f(w_0, \gamma, M, k)$$
$$= g(w_0, \gamma, M, k) \tag{15}$$

where w_0, the local thickness/incidence function, is evaluated at the ith panel. The functions f and g are uniformly valid solutions, derived from the composite Cp's of Eq. (12), for compression and expansion surfaces, respectively. It is clear that the matrix $[E]$ only contains elements that are self-influenced, hence quasi-steady in terms of the flow history.

The underlying principle for the composite Cp due to compression wave is that its departure from the exact hypersonic compression Cp amounts to the difference between the third-order series of Donov[13] and that of Linnell[17]; whereas for the composite Cp due to expansion wave is that its departure from the exact simple wave solution amounts to the difference between the third-order series of Busemann[12] and that of Lighthill's piston theory.[2] The parameter μ of Eq. (14) is a local switching operator on matrix $[E]$ for suitable selection of function f or g on each panel. It should be noted that Eq. (14) contains piston theory as a special case, in which $[D] = 0$, $\mu = 1$ leading to

$$E_{ii}^{-1} = (4/M) + 2(\gamma + 1)w_0 + (\gamma + 1)Mw_0^2 \tag{16}$$

In the Newtonian limit, where M approaches infinity and γ approaches unity simultaneously, E_{ii}^{-1} reduces, as expected, to

$$E_{ii}^{-1} = f(w_0) = 2w_0^2 \quad \text{and} \quad E_{ii}^{-1} = g(w_0) = 0 \tag{17}$$

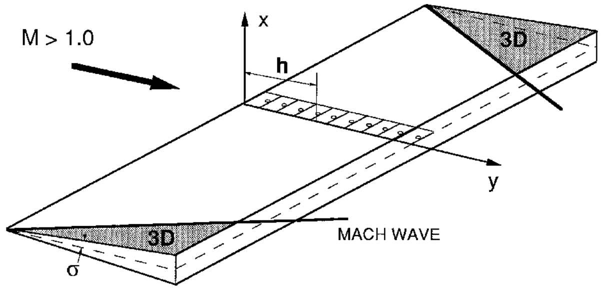

Fig. 2 Rectangular wing model with wedge profile in supersonic flow showing a two-dimensional chordwise strip.

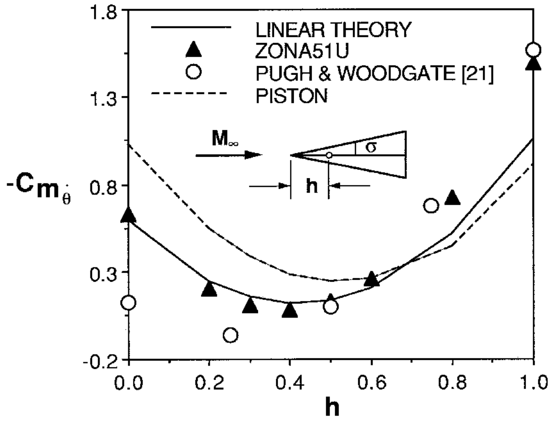

Fig. 3 Damping-in-pitch derivative for a wedge profile vs pitching axis location: M = 1.75, σ = 6.85 deg.

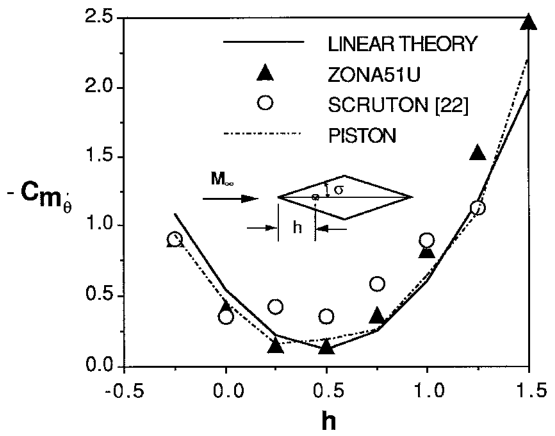

Fig. 4 Damping-in-pitch derivative for a wedge profile vs pitching axis location: M = 2.43, σ = 6.85 deg.

Applications

The present unified hypersonic–supersonic lifting surface method has been fully developed into a computer program now known as the ZONA51U code (U stands for unified). In what follows, the present method is applied to several typical cases for aeroelastic applications. These include rigid wedge in pitching motions, leading-edge flap oscillation, panel flutter, and flutter analysis for wing planforms. Unsteady pressures, stability derivatives, generalized forces, and flutter boundaries are presented for these cases. Comparison with results of Hui's exact theory,[20] Van Dyke's second-order theory,[11] PEC method,[8] piston theory,[2,3,9,10] linear theory[4] (or ZONA51), and available measured data are shown wherever appropriate.

The lift and moment slope coefficients are defined, respectively,

$$C_{L_\alpha} = \frac{1}{A}\int_A \Delta C_p \,\mathrm{d}x\,\mathrm{d}y$$

$$C_{M_\alpha} = \frac{1}{Ac}\left[\iint_A \Delta Cp(x - x_0)\,\mathrm{d}x\,\mathrm{d}y + \int_A \Delta Cp\left(z\,\frac{\mathrm{d}z}{\mathrm{d}x}\right)\mathrm{d}x\,\mathrm{d}y\right] \tag{18}$$

where $z = z(x, y_i)$ is the sectional airfoil thickness distribution. The stiffness and damping moments are related to these expressions for vanishing values of k, i.e.,

$$C_{m_\theta} = \mathrm{Re}(C_{m_\alpha}) \quad \text{and} \quad C_{m_{\dot\theta}} = \mathrm{Im}(C_{m_\alpha})/k \tag{19}$$

A rectangular wing model, with the wedge profile containing a two-dimensional flow at the inboard sections, is shown in Fig. 2. Computed results of ZONA51U from the root chord strip is used to verify with those provided by other two-dimensional theories (Figs. 3–12).

Oscillating Wedge/Diamond Profiles

Effect of Pitching Axis Location

In Figs. 3 and 4, results of ZONA51U in damping-in-pitch derivative are compared with those of linear theory

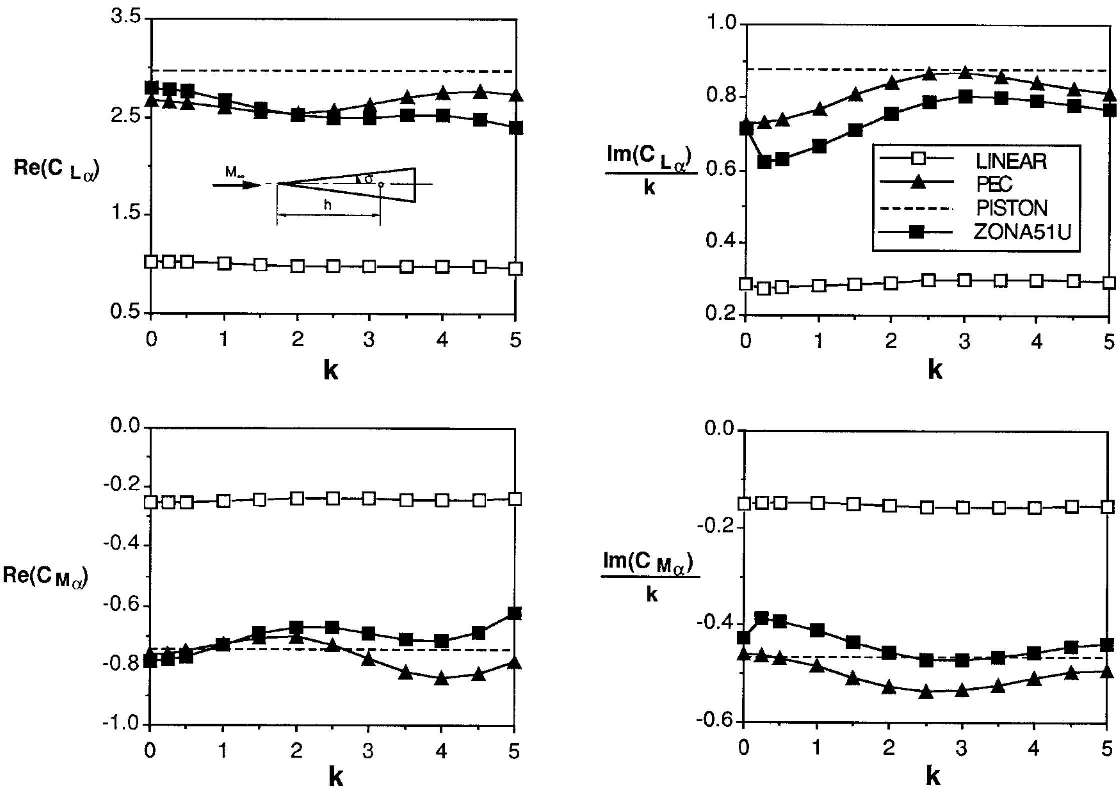

Fig. 5 Stability derivatives for an oscillating wedge vs reduced frequency: M = 3.0, h = 0.25c, σ = 10 deg.

(ZONA51), piston theory, and test data[21,22] for a wedge profile and a diamond profile, respectively. It is seen that ZONA51U predicts a closer trend to the test data than do piston theory and linear theory.

Effect of Reduced Frequency

Figure 5 presents the variations of the lift and the moment coefficients C_{L_α} and C_{m_α} with reduced frequency k for an oscillating wedge. Results of ZONA51U are compared with those of linear theory and PEC method. Good correlation is found between results of ZONA51U and PEC at $M = 4.0$ for a wedge of semiangle $\sigma = 15$ deg, whereas substantial departures are found between that of linear theory and ZONA51U. Clearly, these departures represent the additional nonlinear thickness effect to the results of linear theory.

Effect of Mach Number

Figure 6 presents the variations of stiffness and damping-in-pitch moments with freestream Mach numbers for a diamond profile of thickness ratio $\tau = \tan(15 \text{ deg})$. It is seen that results of the stiffness moment of ZONA51U follows the resulting trend of Hui's exact theory[20] throughout the Mach range, whereas those of piston theory and linear theory remain independent of Mach number. Considerable departures between various results are found for the damping moment beyond $M = 4.0$. Methods due to Van Dyke's theory[11] and piston theory yield diverged results at the hypersonic end, whereas the results of ZONA51U are in good agreement with Hui's exact theory[20] throughout the Mach number range, showing their uniform validity. In fact, only the latter two theories approach the proper Newtonian limit.

Effect of Thickness

Figures 7 and 8 present the variations of the stability derivative for a diamond profile with profile thickness at $M = 2.0$,

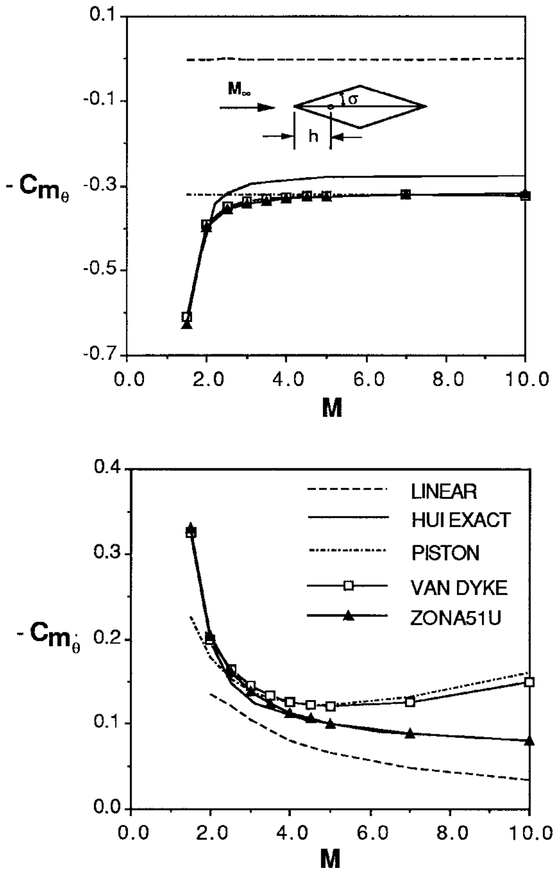

Fig. 6 Stiffness and damping-in-pitch derivatives for a diamond profile vs Mach number: $h = 0.5c$, $\sigma = 15$ deg.

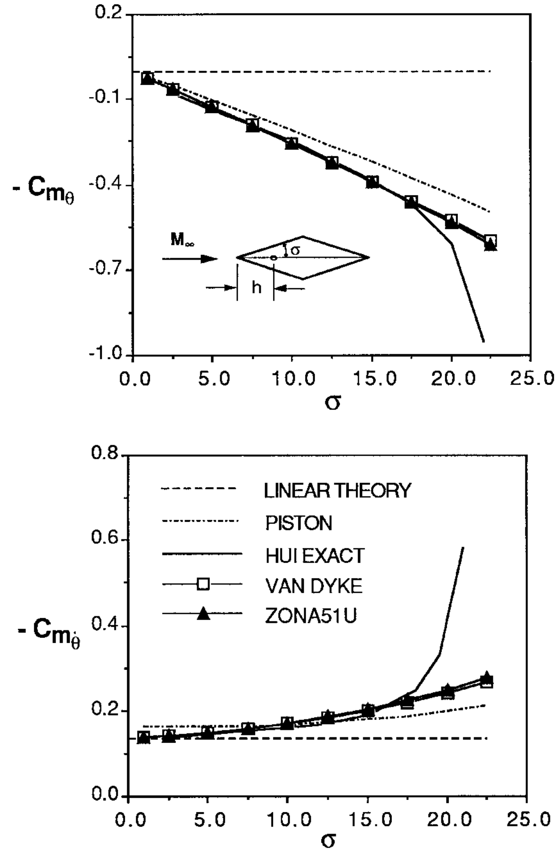

Fig. 7 Stiffness and damping-in-pitch derivatives vs semiwedge angle: $M = 2.0$, $h = 0.5c$.

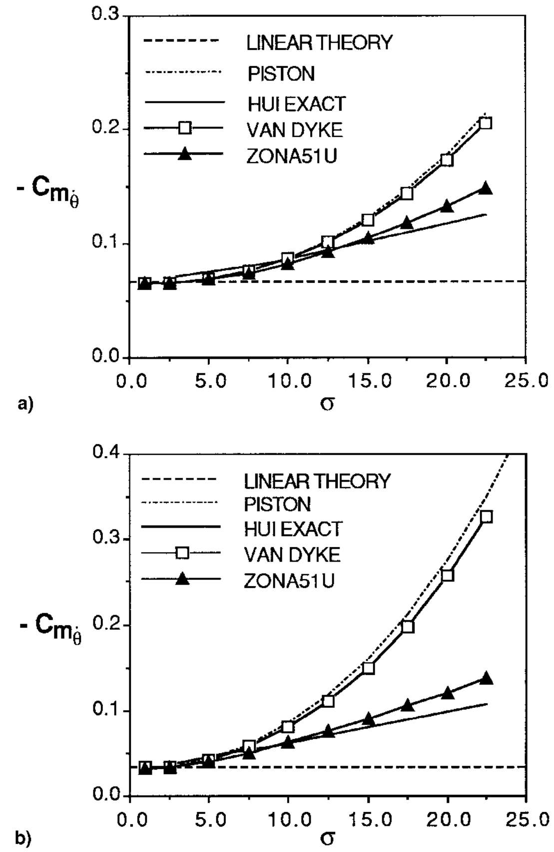

Fig. 8 Damping-in-pitch derivative vs semiwedge angle, $h = 0.5c$: M = a) 5.0 and b) 10.0.

5.0, and 10.0, respectively. Figure 7 shows that both linear theory and piston theory underpredict the stiffness and damping moments; whereas good agreement is found between ZONA51U and Hui's exact theory[20] up to $\sigma = 15$ deg. When Mach number is increased to $M = 5.0$ and 10.0, Van Dyke's theory[11] and piston theory overpredict the damping moment. ZONA51U, however, yields results in close agreement with Hui's exact theory[20] up to $\sigma = 15$ deg for all Mach numbers considered. Thus, given pitching axis location at half chord, an increase in thickness results in an increase in damping moment. The sudden increase in damping moment of Hui's exact theory[20] at $M = 2.0$ depicts the shock detachment occurring around $\sigma = 23.5$ deg. Such a trend is beyond the capability of the current version of ZONA51U. Similar to piston theory, ZONA51U's nonlinear thickness effect is accounted for only by a quasi-steady approach, which ignores the flow history. However, imbedded in ZONA51U, the linear theory (ZONA51) models the flow with Mach waves in the complete frequency domain, thereby conveying the three-dimensional unsteady upstream influence properly. Hui's exact theory[20] accounts for mildly unsteady flow, hence the unsteady shock and Mach wave interaction that includes the effect due to shock detachment.

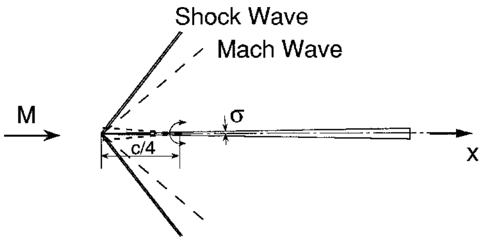

Fig. 9 Oscillating leading-edge flap of a thin-wedge airfoil: σ = 3 deg.

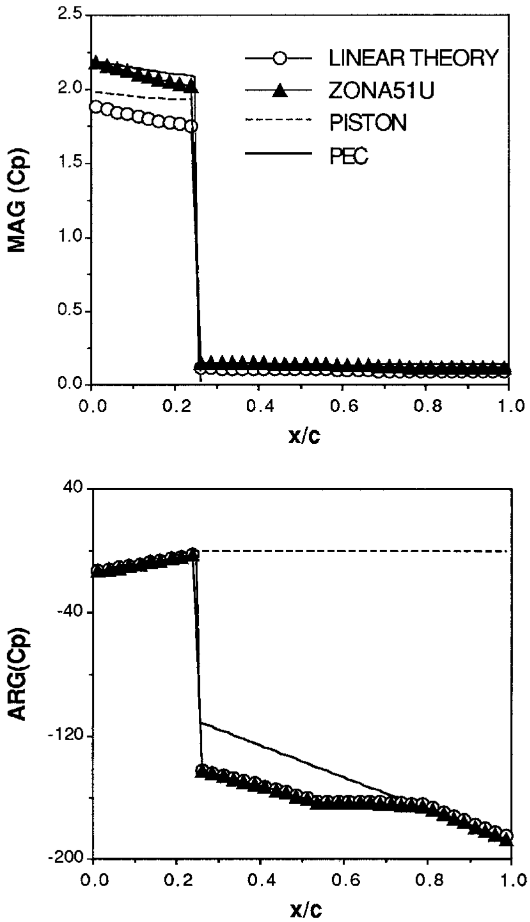

Fig. 10 Unsteady pressure distributions for an oscillating leading-edge flap with hinge line at quarter chord: $M = 2.4$, $k = 0.5$, $\sigma = 3$ deg.

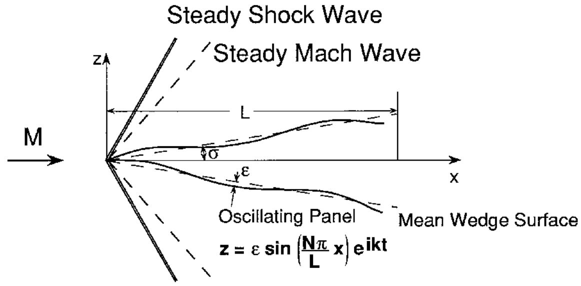

Fig. 11 Oscillating panels mounted on a wedge with semiwedge angle σ = 3 deg.

Leading-Edge Flap Oscillation

Figure 9 shows an oscillating leading-edge flap of thin wedge profile ($\sigma = 3$ deg) with a hinge line located at quarter chord. Figure 10 shows the magnitude and phase angle of unsteady pressures at $M = 2.4$, $k = 0.5$. The parameter $kM\tau$ is bound by 0.1 in this case. It is seen that not only piston theory underestimates the pressure magnitude, but it predicts zero-phase change downstream of the hinge line.

Significant improvement over linear theory by the results in ZONA51U is found in the pressure magnitude on the flap. As expected, little improvement is found between their resulting phase angles. The difference between the results of PEC and ZONA51U lies in the inadequacy of ZONA51U in accounting for the effect of unsteady shock/Mach wave interaction, and thus, the effects of rotationality and flow history.

Panel Flutter

Figure 11 shows two flexible panels (membranes) mounted on both surfaces of a wedge ($\sigma = 3$ deg). The panels are performing oscillatory mode as depicted by

$$H_n(x, t) = \varepsilon h_n(x) e^{ikt}, \qquad h_n = \sin[(n\pi/L)x] \tag{20}$$

where $n = 1, 2$, and ε is the amplitude of vibration.

Figure 12 presents the effect of reduced frequency on generalized aerodynamic forces Q_{ij} for these vibrating panels at $M = 4.0$, where Q_{ij} is defined as

$$Q_{ij} = \int_A \Delta Cp_j h_i \, dx \, dy, \qquad (i = 1, 2 \quad \text{and} \quad j = 1, 2) \tag{21}$$

Results in Q_{12} and Q_{21} of linear theory, piston theory, and ZONA51U are compared with that of PEC. Similar to the earlier observation in the case of flap oscillation, ZONA51U in Fig. 12 substantially improves the pressure magnitude over that of linear theory, but it improves little in phase angles, as expected.

Wing Flutter

Two wing planforms are selected for performing flutter analysis using ZONA51U: a 70-deg delta wing and a 15-deg swept untapered wing.

70-Deg Delta Wing

Figure 13 presents flutter boundaries for a 70-deg delta wing with a 6% thick biconvex airfoil section. The normalized flutter speed is defined as $V_f/(b\sqrt{\mu}\omega_2)$, where b is the semichord at 3/4 semispan, μ is the mass ratio, and ω_2 is the natural frequency of the first torsion mode.

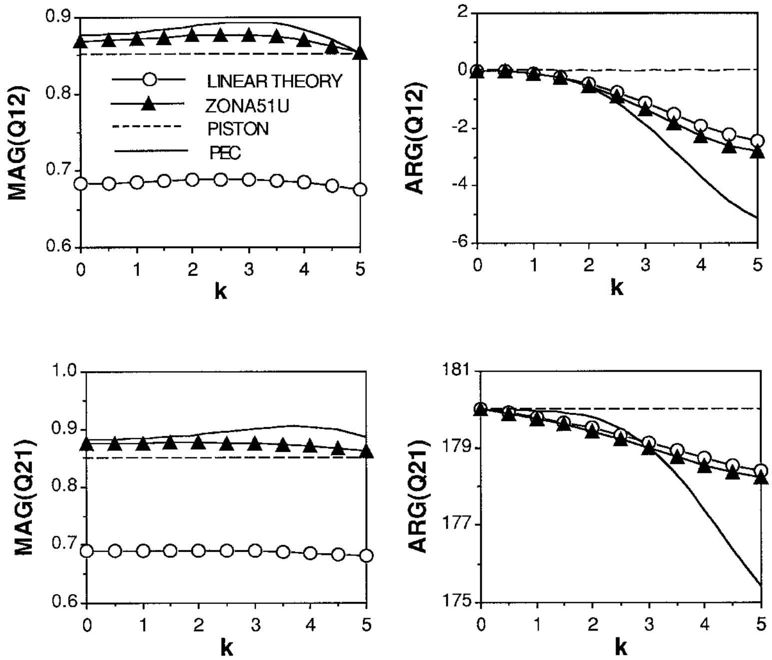

Fig. 12 Effect of reduced frequency on general aerodynamic forces for an oscillating panel: M = 4.0, σ = 2 deg.

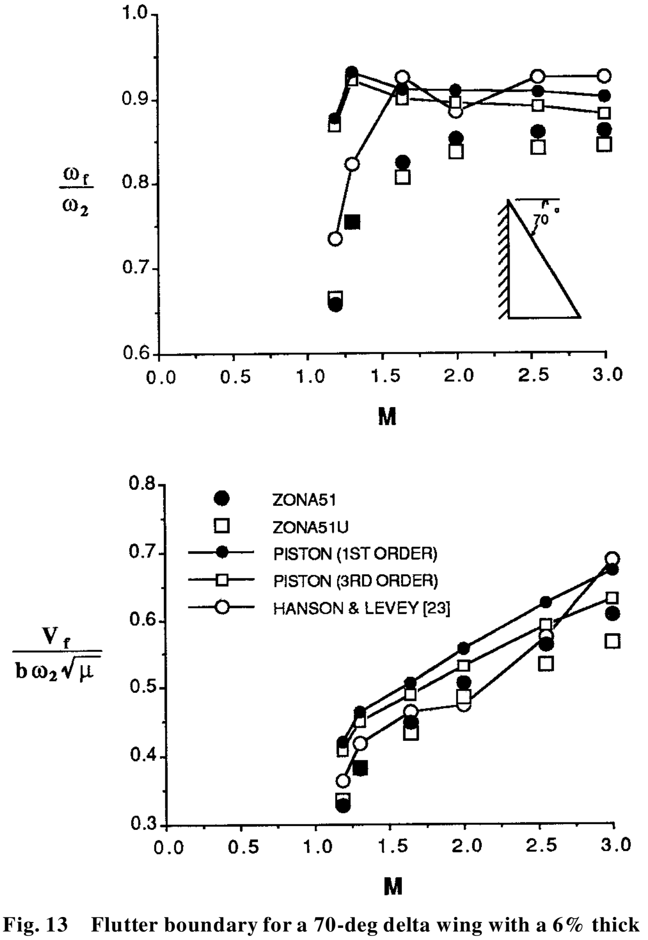

Fig. 13 Flutter boundary for a 70-deg delta wing with a 6% thick biconvex airfoil section.

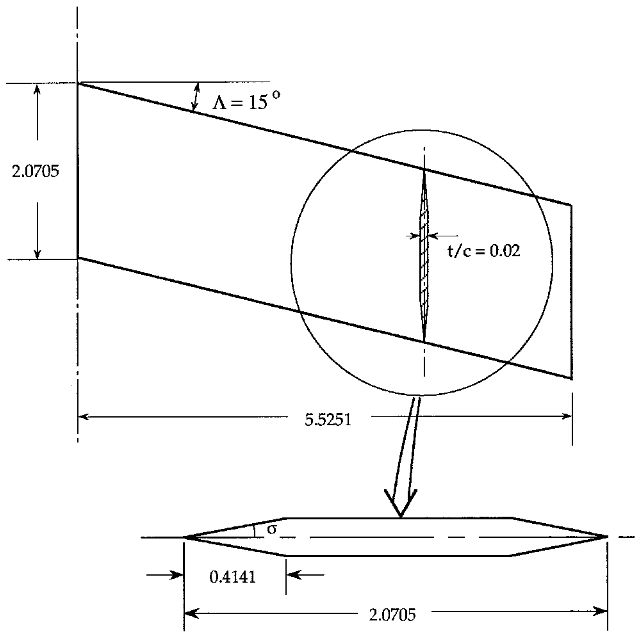

Fig. 14 15-Deg swept untapered wing showing dimensions. tan(σ) = 0.05 and units are in inches.

The flutter experiment was carried out at the NASA Langley Research Center by Hanson and Levey.[23] The wing model used was essentially a flat plate. According to Ref. 23, four measured modes are used in the present flutter analysis. Half of the delta planform is subdivided into 10 × 10 panels. The flutter boundary consists of the flutter points obtained for six Mach numbers (M = 1.19, 1.30, 1.64, 2.0, 2.25, and 3.0) using ZONA51 and ZONA51U. Flutter results computed by piston theory, ZONA51, and 51U are compared with the measured data.[23] Several observations on the performance of ZONA51U are as follows:

1) ZONA51U and the third-order piston theory predict more conservative flutter boundaries than that of ZONA51 and the first-order piston theory, respectively, indicating that the thickness effect indeed reduces flutter speed in supersonic flight.

2) ZONA51U predicts the most conservative flutter boundary of all methods considered.

3) Similar flutter trends are found between results here and that of a 6% diamond airfoil section,[24] demonstrating that similar airfoil section shapes have an insignificant impact on the flutter boundary.

Table 1 Flutter speed and frequency for a 15-deg swept untapered wing

Methods	M = 1.3, aluminum wing, ρ_r = 0.00049 slug/ft^3		M = 3.0, magnesium wing, ρ_r = 0.00093 slug/ft^3	
	V_f, ft/s	f_f, Hz	V_f, ft/s	f_f, Hz
Test	1280	102	2030	146
Rodden[10]	1397	124	1913	149
ZONA51	1547	127	2170	148
ZONA51U	1397	119	1805	147

15-Deg Swept Untapered Wing

Table 1 presents three computed flutter points for a 15-deg swept untapered wing of aspect AR = 5.35 at M = 1.3 and 3.0, where f_f denotes the flutter frequency ($f_f = \omega_f/2\pi$).

The flutter experiment was carried out at the NASA Langley Field by Tuovila and McCarty.[25] The wing model used is a cantilever wing with a 2% thick hexagonal airfoil section (Fig. 14). According to Ref. 25, eight modes generated by MSC/NASTRAN are used in the present flutter analysis. Half of the wing planform is subdivided into 10 × 10 panels.

In Table 1, computed results of ZONA51, Rodden's method[26] (employing ZONA51), and ZONA51U are compared with test data of Tuovila and McCarty.[25] Although the computing time of Rodden's method[26] is comparable to that of ZONA51U, the applicability of Rodden's method[26] is confined to supersonic Mach numbers. Here, ZONA51U once again yields the most conservative flutter speed at both Mach numbers. By contrast, linear theory yields the most nonconservative flutter speed.

Conclusions

A unified lifting surface method has been developed that can account for the wing thickness and/or flow incidence effects at all supersonic and hypersonic Mach numbers.

The concept of piston theory is generalized to yield a nonlinear thickness AIC matrix which is suitably integrated with the supersonic linear AIC. Thus, in contrast to piston theory, the present method can account for the effects of upstream influence as well as three dimensionality for any given lifting surface system in an unsteady supersonic–hypersonic flow.

Close examination of all existing third-order theories, including piston theory, reveals that they all fail to yield proper limits on either the low supersonic end or the hypersonic end. Here, the established composite function (for compression and expansion waves) is uniformly valid for all supersonic–hypersonic Mach numbers. Thus, it extends the method applicability to cover the Ackeret limit as well as the Newtonian limit at both ends.

Various cases studied lead to the conclusion that ZONA51U makes a substantial improvement over linear theory and piston theory in terms of the resulting unsteady pressures, stability derivatives, and generalized forces. In particular, piston theory is found to be inadequate for present cases in predicting flap aerodynamics and panel flutter, whereby it provides no phase changes whatsoever. Since the nonlinear matrix only corrects the thickness effect, but not the flow history, the present computed phase angle is expected to follow that of linear theory. In terms of pressure magnitude and stability derivatives, the present results are expected to be in better agreement with those of the PEC method. ZONA51U consistently predicts conservative flutter boundaries for a 70-deg delta wing and 15-deg swept untapered wing. Thus, it confirms that the supersonic thickness effect is to reduce the flutter speed.

Overall applicability assessment indicates an estimate thickness limitation to the present method at τ = tan (15 deg) for supersonic Mach numbers up to 10, whereas no input restriction is required of the wing swept, aspect ratio, and other planform parameters. In view of the limitation in available measured data, further validation and applicability assessment of the present method are warranted. Finally it is remarked that the computing time of ZONA51U is comparable to that of ZONA51. Hence, it should serve as a viable method for high-speed applications such as NASP or HSCT.

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